– Astrophysical Hydrodynamics –
Exam 2023/2024

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3[rd] of April 2024, 15:00-17:00

Important notes

– No internet-connected devices are allowed, no notes nor books are allowed.
– Write your name and student number on *all* the pages that you hand in.
– Please make sure that your handwriting is easy to read!

This exam consists of 2 concise questions and 2 exercises.
Each question is worth a maximum of 1.5 points (0.5 per subquestion), exercise 1 and 2 are worth a maximum of 3 and 4 (1 per item) points, respectively. The final grade of the exam is the sum of all the points.

Tips:
– Give concise answers to the questions, if not requested do not embark on derivations of equations.
– Read the text of the questions and exercises very carefully.
– If, as a result of a calculation, you obtain values that do not make sense to you but you cannot find the error, write a comment indicating the approximate value that you would have expected and why.

Questions

Q1

The explosion of a massive star as a supernova produces a strong shock that propagates in the interstellar medium (ISM) of a galaxy forming a supernova remnant.

(i) Following a very short phase of free expansion, the supernova remnant enters the Sedov phase. Can you describe two physical properties (e.g. temperature and density) of the medium that is perturbed by the shock during this phase?

(ii) As the Sedov phase ends and the supernova remnant enters the radiative phase, how do the previous properties of the perturbed gas change?

(iii) If the ISM where the supernova explodes is significantly more turbulent than the one of the Milky Way, thus with a turbulent velocity dispersion significantly larger that $10\,\mathrm{km\,s^{-1}}$, would you expect the supernova remnant to be more or less efficient at transferring kinetic energy to the ISM? Why?
Tip: Efficiency is measured as the percentage of kinetic energy transferred to the ISM relative to the supernova's initial energy.

Q2

A cloud of molecular gas has a mass that is one order of magnitude larger than the Jeans mass of gas with its same density and temperature.

(i) Describe the dynamical state of the cloud and its evolution (if applicable), assuming gravity is only counteracted by thermal pressure.

(ii) How might the presence of a magnetic field within the molecular cloud alter this scenario?

(iii) Considering the cloud might either collapse to form a star or disperse into the surrounding ISM, in which of these scenarios would you need to introduce an additional term on the right-hand side of the continuity equation to describe the ISM? What type of term would this be?

Exercises

E1

A galaxy is surrounded by a large atmosphere of hot gas having a temperature $T_\mathrm{h} = 2\times10^6\,\mathrm{K}$, a metallicity $[\mathrm{Fe/H}] = -1.0$, and an average number density $n_\mathrm{h} = 1\times10^{-3}\,\mathrm{cm^{-3}}$. Consider that a portion of this gas (roughly spherical in shape) undergoes a thermal perturbation that makes its temperature decrease to $2\times10^5\,\mathrm{K}$, while *remaining in pressure equilibrium* with the environment. The diameter of the perturbed portion is $D = 5\,\mathrm{kpc}$ and we aim to estimate its further evolution. There are two competing phenomena at play: thermal conduction from the hot medium tends to bring the portion of gas back to its original temperature, while radiative cooling tends to make it cool further, leading to a thermal instability.

1. Assuming the Spitzer treatment and heat exchange dominated by thermal conduction,

one can obtain the following formula for the time scale of thermal conduction:

$$t_{\rm cond} \approx \frac{nk_{\rm B}L^2}{\kappa_{\rm Sp}T_{\rm h}^{5/2}}, \tag{1}$$

where n is the density of the gas, $k_{\rm B}$ is the Boltzmann constant, L is a relevant length scale and $\kappa_{\rm Sp} = 6.1 \times 10^{-7}\,{\rm erg\,s^{-1}\,cm^{-1}\,K^{-7/2}}$. Estimate on what time scale thermal conduction can bring the portion of gas back to the original temperature $T_{\rm h}$.
Tip: Pay attention to the correct gas density and length scale to use.

2. Using the cooling function for collisional ionisation equilibrium (Fig. 1) calculate the cooling time of the portion of thermally perturbed gas and determine which effect between heating by thermal conduction or cooling by radiation will prevail.

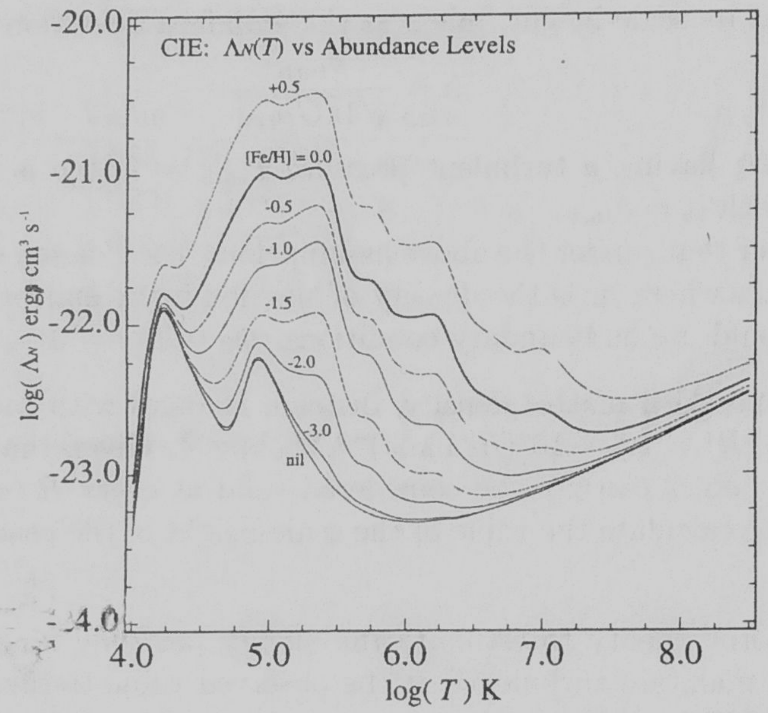

Figure 1: The cooling function $\Lambda(T)$ obtained under the assumption of collisional ionisation equilibrium for different gas metallicities.

3. Assume now that a thermal instability has taken place (cooling has prevailed) and that the gas in the portion has reached a new equilibrium temperature of $2 \times 10^4\,$K while (i) maintaining pressure equilibrium with the hot medium, (ii) remaining largely ionised, and (iii) reducing its size by a factor given by mass conservation. This instability has effectively produced a "cold" cloud embedded in a hot medium. Estimate the Jeans length of gas with the properties of this cloud and discuss the dynamical equilibrium of the cloud, i.e. importance of self-gravity.
Tip 1: Mass conservation means that all the gas in the perturbed portion (mass $\propto \rho D^3$) ends up in the cloud.

Tip 2: If you do not remember the Jeans length, consider that you can always construct it using sound speed and free-fall time.

E2

Consider the gaseous disc of a dwarf galaxy where the gas is in *vertical hydrostatic equilibrium* with an external gravitation potential (Φ) dominated by the dark matter halo (the stellar disc provides negligible contribution). The temperature of the gas is low ($T \sim 10^2\,\mathrm{K}$) but there is significant turbulence with a velocity dispersion $\sigma_{\mathrm{turb}} = 10\,\mathrm{km\,s^{-1}}$, thus the gas pressure is dominated by the turbulence pressure $P_{\mathrm{turb}} = \sigma_{\mathrm{turb}}^2 \rho$, where ρ is the gas density.

1. Assuming that the hydrostatic equilibrium in this galaxy is akin to that of an infinite slab with the same properties and that the dark matter density ρ_{DM} is constant with height (z) in the region of interest, show that the vertical density profile of the gas is a Gaussian function and that its scale-height, taken as the standard deviation of the Gaussian, is

$$h = \frac{\sigma_{\mathrm{turb}}}{\sqrt{4\pi G \rho_{\mathrm{DM}}}}. \qquad (2)$$

   Tip 1: Note that having a turbulent pressure $P_{\mathrm{turb}} = \sigma_{\mathrm{turb}}^2 \rho$ is analogous to have an isothermal gas with $c_s = \sigma_{\mathrm{turb}}$.
   Tip 2: Remember that, under the above assumptions, the Poisson equation along z reads $\mathrm{d}^2\Phi/\mathrm{d}z^2 = 4\pi G \rho_*$, where ρ_* is the density of the dominant matter component and that, as usual, one should use as boundary conditions: $\Phi_0 = \Phi(z=0) = 0$ and $\left(\frac{\mathrm{d}\Phi}{\mathrm{d}z}\right)_{z=0} = 0$.

2. Consider then that dark matter density, instead, changes with radius and, close to the midplane, is $\rho_{\mathrm{DM}}(R) = 1.8 \times 10^8 (R/1\,\mathrm{kpc})^{-2}\, M_\odot\,\mathrm{kpc}^{-3}$. Given that the circular speed is roughly constant, eq. 2 can now be considered valid at every R (σ_{turb} remains constant with R). Use it to calculate the value of the scale-height of the gaseous disc as a function of radius.

3. Use the Kolmogorov theory to estimate the energy per unit time and area that needs to be injected to maintain turbulence at the observed value (represented by the velocity dispersion σ_{turb}). Make this calculation at two locations: (i) in the central part of the disc at $R = 2\,\mathrm{kpc}$ from the centre, and (ii) in the outer disc at $R = 10\,\mathrm{kpc}$. Assume that gaseous disc has a surface density $\Sigma_g = 10^7\, M_\odot\,\mathrm{kpc}^{-2}$ constant with radius.
   Tip: Note that you need the energy rate per *per unit area*, i.e. the surface density of energy injection ($\dot{\Sigma}_{\mathrm{E,turb}}$) while $\dot{\mathcal{E}}$ in the Kolmogorov theory is *specific*. Useful units for $\dot{\Sigma}_{\mathrm{E,turb}}$ are: $\mathrm{erg\,yr^{-1}\,kpc^{-2}}$.

4. Consider two possible sources of turbulence. The first one is supernova explosions that release a total energy per supernova $E_{\mathrm{SN}} = 10^{51}\,\mathrm{erg}$ with a rate of explosions $\Sigma_{\mathrm{SNe}} = 2 \times 10^{-5}\,\mathrm{yr^{-1}\,kpc^{-2}}$ at $R = 2\,\mathrm{kpc}$ and $\Sigma_{\mathrm{SNe}} = 2 \times 10^{-7}\,\mathrm{yr^{-1}\,kpc^{-2}}$ at $R = 10\,\mathrm{kpc}$. The second source is gas infall from the surrounding environment with a rate of kinetic energy $\dot{\Sigma}_{\mathrm{E,in}} = 2 \times 10^{44}\,\mathrm{erg\,yr^{-1}\,kpc^{-2}}$ constant across the disc. Which of these two is more likely responsible for maintaining the turbulence in the two above locations?

Physical constants

Constants	Symbol	Value	Units (cgs)
Speed of light	c	$2.99792458 \times 10^{10}$	$\mathrm{cm\,s^{-1}}$
Gravitational constant	G	6.67408×10^{-8}	$\mathrm{cm^3\,g^{-1}\,s^{-2}}$
Boltzmann constant	k_B	$1.38064852 \times 10^{-16}$	$\mathrm{erg\,K^{-1}}$
Planck constant	h	$6.626070040 \times 10^{-27}$	$\mathrm{erg\,s}$
Mass of electron	m_e	$9.10938356 \times 10^{-28}$	g
Mass of proton	m_p	$1.672621898 \times 10^{-24}$	g
Avogadro constant	N_A	$6.02214179 \times 10^{23}$	$\mathrm{mol^{-1}}$
Elementary charge	e	$4.80320425 \times 10^{-10}$	e.s.u. $\left(\mathrm{erg^{1/2}\,cm^{1/2}}\right)$

Astrophysical standard values

Constants	Symbol	Value	Units
Parsec	pc	3.0856776×10^{18}	cm
Solar mass	$M_\odot$	1.9891×10^{33}	g
Solar radius	$R_\odot$	6.95508×10^{10}	cm
Sideral year	yr	3.155815×10^{7}	s